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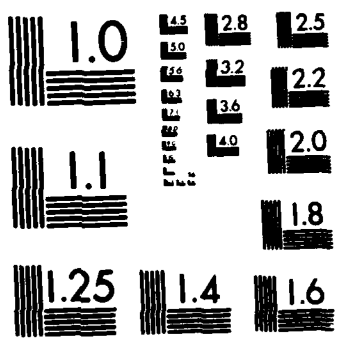

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TECHNICAL REPORT ARLCB-TR-85022

RECURSIVE GRADIENT ESTIMATION USING SPLINES FOR NAVIGATION OF AUTONOMOUS VEHICLES

C. N. SHEN

JULY 1985

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT CENTER
LARGE CALIBER WEAPON SYSTEMS LABORATORY
BENÉT WEAPONS LABORATORY
WATERVLIET N.Y. 12189

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ARLCB-TR-85022	2. GOVT ACCESSION NO. AD-A159883	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) RECURSIVE GRADIENT ESTIMATION USING SPLINES FOR NAVIGATION OF AUTONOMOUS VEHICLES		5. TYPE OF REPORT & PERIOD COVERED Final
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) C. N. Shen		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS US Army Armament Research & Development Center Benet Weapons Laboratory, SMCAR-LCB-TL Watervliet, NY 12189-5000		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS AMCMS No. 6111.01.91A0.011 PRON No. 1A425M511A1A
11. CONTROLLING OFFICE NAME AND ADDRESS US Army Armament Research & Development Center Large Caliber Weapon Systems Laboratory Dover, NJ 07801-5001		12. REPORT DATE July 1985
		13. NUMBER OF PAGES 38
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Presented at Second Army Conference on Applied Mathematics and Computing, Rensselaer Polytechnic Institute, Troy, NY, 22-24 May 1984.
Published in Conference Proceedings.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Terrain Slope Estimation
Autonomous Vehicle Navigation
Two-Dimensional Recursive Smoothing
Polynomial Splines
Laser Range Matrix

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Terrain gradient estimation is needed for navigation of an autonomous vehicle in climbing the hills. The in-path and cross-path terrain slopes are estimated from the set of corresponding range slopes. A two-dimensional recursive smoothing algorithm using polynomial splines in the third dimension is developed for this purpose. Approximations are introduced in the sub-optimal system so that the computation time increases only linearly with the size of the two-dimensional data.

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INTRODUCTION

The successful development of an autonomous vision system for mobile vehicles would be of considerable value and importance to defense and related fields. Numerous reports and studies currently recommend artificial intelligence/robotics applications which require autonomous vehicles. Essential to these robotic vehicles is an adequate and efficient computer vision system. A potentially more successful approach, other than TV pictures and photographs, would be to develop a three-dimensional system employing a laser rangefinder.

A range matrix describing a certain scanned area of the terrain in front of the mobile robot can be used to estimate the slopes of the terrain. The in-path and cross-path slopes of the terrain are evaluated by a slope estimation scheme. These slope informations along the passible corridors are utilized to determine a safer and more accurate path for the mobile robot vehicle to travel.

The mobile robot vehicle is equipped with data acquisition and decision making devices for its autonomous navigation over rough terrain. A laser rangefinder can be operated by emitting laser pulses and measuring the time of flight of a pulse between the instant it is transmitted and the instant the reflected pulse is received. This time of flight is related to the distance between the transmitter and the point on the terrain from which the pulse is reflected. The terrain is scanned by changing the azimuth and elevation angles of the laser beam in a discrete fashion. The measurements are then available in the form of an NXM 'range-matrix'.

References are listed at the end of this report.

The slope estimation problem dealt with here is one of obtaining smoothed estimates of function values and particularly their derivatives from a finite set of inaccurate measurements in two dimensions. In one approach we can identify the dynamic equations of the underlying system, or estimate the distributions for the quantities of interest and then apply optimal estimation algorithms. In some engineering problems the stochastic system may not be identified easily and in these situations, spline smoothing has proved to be a useful alternative.

In this report, we obtain the smoothed estimates of the slopes by utilizing a two-dimensional smoothing algorithm. For the problem of smoothing a finite set of noise corrupted data of an unknown function, it is proposed to obtain the smoothed estimate by fitting a two-dimensional approximating function to the data set, for a set of measurements corrupted by a white noise process.

HISTORICAL REVIEW

By noting the fact that original signals such as visual scenes are in analog form, techniques were developed which reconstruct analog signals from discrete data by utilizing interpolation or approximating functions. Frequency domain interpretation of the interpolation process was reported in Reference 1. Also, B-spline interpolates (refs 2-4) were used (ref 5) in restoring a continuous signal from a set of digitized data. For one-dimensional noise corrupted data generated by unknown systems, Reinsch (ref 6) utilized natural cubic splines (refs 2-4) along with least squares constraints to solve the problem of curve plotting. Hou and Andrews (ref 7) constructed

continuous-discrete image and utilized spline basis functions along with the least squares constraints for image restoration. Because of their non-recursiveness, the algorithms in References 6 and 7 are involved with complex computations and cannot be implemented on-line. Recently, by using a reproducing kernel Hilbert space approach, Weinert et al (refs 8,9), developed a structural correspondence between spline interpolation and linear least squares smoothing of a particular random process.

In recent years, two-dimensional recursive filters have drawn much attention because of the need for processing images or other two-dimensional information. Previous efforts (refs 10-12) to achieve a truly recursive two-dimensional filter were of only limited success because of the difficulty in establishing a suitable two-dimensional recursive model as well as the high dimension of the resulting matrix and state vector. Recently, by using a two-dimensional recursive model obtained from a two-dimensional spectral factorization technique (ref 13), Woods and Radewan (ref 14) developed a two-dimensional Kalman vector processor and a two-dimensional Kalman scalar processor.

The above mentioned time-domain design techniques assumed or identified a two-dimensional stationary discrete system model at the beginning of their problem formulation. On the other hand, Reinsch (ref 6) interpreted a one-dimensional data smoothing problem as an optimal curve-fitting problem arising in approximation theory, and proposed a nonrecursive smoothing algorithm using smoothing splines. For a two-dimensional image restoration problem, Hou and Andrews (ref 7) followed the approach taken by Reinsch (ref 6), and extended it to a two-dimensional problem in a nonrecursive manner. In this report, we

develop a two-dimensional recursive smoothing algorithm. Compared to its nonrecursive counterpart, this recursive algorithm will require less computational complexity and memory space. Especially, the amount of computation needed at each iteration is independent of the size of the two-dimensional data.

PROBLEM FORMULATION FOR ONE-DIMENSIONAL APPROXIMATION

From the viewpoint of approximation theory, when a set of discrete observation data is noise free, spline interpolation provides a means of optimally reconstructing an unknown original signal. When the observation data are corrupted by noises, and if the form of the original continuous signal is known, then we can use least squares estimation techniques to approximate the original signal. We are now dealing with a problem in which an unknown signal is approximated by smoothing splines from a set of noise corrupted observation data. Specifically, an unknown signal $f(\xi)$ is approximated by a polynomial spline $s(\xi)$ which minimizes the objective function:

$$J^* = \sum_{n=1}^{N} [s(\xi_n) - m_n]^T R_n^{-1} [s(\xi_n) - m_n] + \{\sum_{n=2}^{N} \rho_n \int_{\xi_{n-1}}^{\xi_n} [s^k(\xi)]^2 d\xi\} \qquad (1)$$

where

m_n is observation data;

$m_n = f(\xi_n) + v_n$, for $n = 1, 2, \ldots, N$;

v_n is a white observation noise process with error covariance R_n;

$R_n \overset{\Delta}{=} E\{v_n \cdot v_n^T\}$;

$\rho_n > 0$ is a smoothing parameter; and

s^k is the k^{th} derivative of $s(\xi)$.

At this point, it is worthwhile to note the physical role of the smoothing parameter ρ_n as follows: (1) when ρ_n becomes very small, $\rho_n \to 0^+$, the resultant approximating function will pass through each data point and become an interpolation function; (2) when ρ_n assumes a very large value, $\rho_n \to \infty$, minimization of the objective function in Eq. (1) corresponds to fitting a straight line to a data set using least squares criterion. Thus, it can be said that the smoothing parameter controls resolution in a tradeoff of the smoothness of the restored function.

<u>Choice of Approximating Function</u>

As has been noted, it is desired to develop a recursive algorithm whose results are sufficiently close to those obtained by directly minimizing the global problem as given by the criterion in Eq. (1). Fundamental problems encountered in developing a recursive algorithm which generates approximating functions are:

1. feasibility of recursive structures,
2. feasibility of numerical calculations.

Regarding the first problem, it has been noted from References 1 through 5 that some of the approximating functions such as polynomial splines and piecewise Hermite polynomials have finite support. That is, a resultant approximating function for one section is mostly affected by its neighboring data points. Thus, a recursive structure with one or more sample delays would result in sufficiently close results to nonrecursive ones.

For the second point, out of a certain set of functionals, an optimal solution to Eq. (1) is an L-spline (refs 2-4). L-spline is a piecewise polynomial of degree 2k-1, and has 2k-2 continuous derivatives in the region

$[\xi_1, \xi_N]$. Here, we propose to restrict our approximating functions to piecewise Hermite polynomials (ref 3) of degree 2k-1, which have k-1 continuous derivatives in the region $[\xi_1, \xi_N]$. Advantages in using piecewise Hermite polynomials are as follows. Define

$$x_i \overset{\Delta}{=} [s(\xi), s'(\xi), \ldots, s^{k-1}(\xi)]^T \Big|_{\xi=\xi_i}, \quad i = 1, \ldots, N$$

Then a piecewise Hermite polynomial $s(\xi)$ is completely determined by x_i, $i = 1, 2, \ldots, N$. For the purpose of clarity in discussion, only the case of k=2 is treated in the following. A piecewise cubic Hermite polynomial is represented as:

$$s(\xi) = \begin{array}{lll} s_{1,2}(\xi) & \text{for} & \xi_1 \leq \xi \leq \xi_2 \\ \vdots & & \vdots \\ s_{N-1,N}(\xi) & \text{for} & \xi_{N-1} \leq \xi \leq \xi_N \end{array} \tag{2}$$

where

$$s_{k-1,k}(\xi) = [\phi_{k,1}(\xi)\psi_{k,1}(\xi)\phi_{k,0}(\xi)\ \psi_{k,0}(\xi)][x_k^T, x_{k-1}^T]^T \tag{2a}$$

$$x_k = [s(\xi_k)\ s'(\xi_k)]^T, \ k = 1, \ldots, N \tag{2b}$$

$$\phi_{k,1}(\xi) = (\xi-\xi_{k-1})^2[(\xi_k-\xi_{k-1}) + 2(\xi_k-\xi)]/(\xi_k-\xi_{k-1})^3 \tag{2c}$$

$$\psi_{k,1}(\xi) = (\xi-\xi_{k-1})^2(\xi-\xi_k)/(\xi_k-\xi_{k-1})^2 \tag{2d}$$

$$\phi_{k,0}(\xi) = (\xi_k-\xi)^2[(\xi_k-\xi_{k-1}) + 2(\xi-\xi_{k-1})]/(\xi_k-\xi_{k-1})^3 \tag{2e}$$

and

$$\psi_{k,0}(\xi) = (\xi-\xi_{k-1})(\xi_k-\xi)^2/(\xi_k-\xi_{k-1})^2 \tag{2f}$$

Smooth Integrals as Quadratics at Node Points

Thus, it becomes natural that the smoothing integral in Eq. (1) is expressed in terms of x_i's, $i = 1, 2, \ldots, N$. With some manipulations in algebra, the smoothing integral for k = 2 in Eq. (1) is represented in a quadratic form as derived in Appendix A.

$$\int_{\xi_{n-1}}^{\xi_n} ||s''(\xi)||^2 \, d\xi = (x_n - A^* x_{n-1})^T B^{-1} (x_n - A^* x_{n-1})$$

$$= \begin{bmatrix} x_{n-1} \\ x_n \end{bmatrix}^T \cdot \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \cdot \begin{bmatrix} x_{n-1} \\ x_n \end{bmatrix} \tag{3}$$

where

$$x_n \overset{\Delta}{=} [s(\xi),\ s'(\xi)]^T \Big|_{\xi=\xi_n} \tag{4}$$

$$A^* = \begin{bmatrix} 1 & \Delta \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} \frac{\Delta^3}{3}, & \frac{\Delta^2}{2} \\ \frac{\Delta^2}{2}, & \Delta \end{bmatrix} \tag{5a}$$

$$\Delta = \xi_n - \xi_{n-1}, \quad \text{for } n = 2,\ldots,N \tag{5b}$$

$$C_{11} \overset{\Delta}{=} A^{*T} B^{-1} A^* \quad , \quad C_{12} \overset{\Delta}{=} C_{21}{}^T \overset{\Delta}{=} -A^{*T} B^{-1}, \quad C_{22} \overset{\Delta}{=} B^{-1} \tag{5c}$$

A nonrecursive solution for the minimization of the objective function in Eq. (1) can be obtained by taking the gradient of J^* with respect to $[x_1, x_2,\ldots,x_N]^T$ and setting it to zero. However, this approach will require solutions of a set of 2N simultaneous equations. To avoid this computational problem, we have developed a recursive algorithm which requires inversion of 2 x 2 matrices only.

RECURSIVE ALGORITHM

Given a set of initial values for the mean $\hat{x}_1$ and its error covariance P_1, where $P_1 = E\{(x_1 - \hat{x}_1)(x_1 - \hat{x}_1)^T\}$, by using Eqs. (3) and (4), the objective function in Eq. (1) becomes

discrete edges, a two-dimensional smoothing algorithm is utilized to estimate the range slopes; (3) estimated range slopes are transformed into terrain slopes.

Estimated Terrain In-Path Slope

The simulation of terrain with hills and valleys is given in Figure 2. The estimated terrain in-path slopes (ref 18) are displayed in terms of a slope map, Figure 2. Characters A,...,G represent a particular range of the terrain in-path slopes increasing from A to G, at the corresponding location. U represents undefined slopes. In Figure 3, we note circular slope regions on the faces of sinusoidal hills and valleys. Also, along a radial direction, the estimated slopes are changing slowly from one region of slopes to another. The large empty spaces are due to the hidden regions at the back of boulders or hills where laser rays could not reach. The undefined gradient represented by 'U' occurs when the recursive algorithm cannot be applied due to sharp changes in ranges between adjacent measurement data. The estimated in-path terrain slope maps are used for the evaluation of the terrain in front of the mobile robot vehicle.

Terrain Cross-Path Slopes

In discussing the terrain cross-path slopes (ref 19), the data can be conveniently processed to generate smoothed in-path and cross-path range slopes recursively in a spherical coordinate system due to the regularity of the elevation and azimuth angles. When we proceed to calculate the true terrain slopes on the base plane, the regularity of the data points is completely destroyed. For a fixed elevation angle β, the horizontal projection of the range data is not located at a fixed distance from the

measurement data in cylindrical coordinates. However, there is a major difficulty in this approach. Even though the two independent variables β_i and θ_j for the rangefinder are changing with constant increments $\Delta\beta$ and $\Delta\theta$, respectively, the independent variable ρ_i in a cylindrical coordinate changes irregularly. The recursive smoothing algorithm in the previous subsection requires that the data points be located at the corners of rectangular grids of the two independent variables. Since the two independent variables ρ_i and θ_j in a cylindrical coordinate system do not form rectangular grids, the smoothing algorithm cannot be applied directly. By noting that the positioning angles β_i and θ_j are changing in regular fashion, it is proposed to obtain the smoothed estimates of the range slopes $dr/d\beta$ and $dr/d\theta$ defined in spherical coordinates. Then, these estimates are transformed to the terrain slopes. In applying the smoothing algorithm to terrain slope estimation, one point to be mentioned is that the basic philosophy of the smoothing spline approach is to suppress the noise elements by fitting a smooth approximating function to a noise corrupted data set. Therefore, when the function to be approximated has sharp changes in its values or derivatives, the smoothing algorithm will produce errors in the results by smoothing out these actual sharp changes. From the viewpoint of terrain slope estimation, such changes occur at the edges of a boulder, a crater, or a ridge on the terrain. Thus, it is proposed to detect these edges by using the rapid estimation scheme. Then, for the area which is free of discrete edges, the two-dimensional smoothing algorithm is utilized to estimate the slopes. The terrain slopes are estimated in the order: (1) discrete edges are detected by using the rapid estimation scheme; (2) for the area which is free of

in the fourth column, $\hat{x}^*_{k,4|k,4}$ and $\hat{x}^*_{k,4|k+1,4}$, and so on.

The approximation method described above is one of the simplest ones. We can employ more elaborate approximation methods at the cost of more complicated computations. By now, we have introduced a recursive quarter-plane processor which computes the filtered estimate $\hat{x}^*_{k,\ell|k,\ell}$ as an approximation to $\hat{x}_{k,\ell|k,\ell}$. From the definition of the estimate $\hat{x}_{k,\ell|p,q}$, the value of $x_{k,\ell}$ which minimizes J is $\hat{x}_{k,\ell|N,M}$. Here, we note that $\hat{x}^*_{k,\ell|k,\ell}$ has its support in the region $R_{(k,\ell)}$, while the nonrecursive solution $\hat{x}^*_{k,\ell|N,M}$ has its support in the region $R_{(N,M)}$. Thus if we desire to have a better approximation to $\hat{x}_{k,\ell|N,M}$, we need to develop a smoothing algorithm which computes $\hat{x}_{k,\ell|k+d_1,\ell+d_2}$ where $d_1,d_2 \geq 1$, $k+d_1 \leq N$ and $\ell+d_2 \leq M$.

The smoothed estimate $\hat{x}_{k,\ell|k+d_1,\ell+d_2}$ is defined as the estimate of $x_{k,\ell}$ obtained by fitting an approximating function in the region $R_{(k+d_1,\ell+d_2)}$. The derivation procedure for the above approximation is similar to that of filtering discussed previously, and is omitted for conciseness.

FURTHER NAVIGATION PROBLEMS

Terrain Slopes and Range Slopes

With reference to Figure 1, terrain in-path and cross-path slopes are defined as the two orthogonal slopes $dz/d\rho$ and $dz/\rho d\theta$ in a cylindrical coordinate system. During the past investigations, the terrain slopes were found to be appropriate measures for evaluating a terrain. A direct approach for estimating the terrain slopes would be to fit a smoothing spline to the

By definition, the filtered estimate $\hat{x}_{k,2|k,2}$ is the estimate of $x_{k,2}$ obtained from an approximating function which minimizes the objective function in Eq. (27) for (p,q) = (k,2). For minimization, we take the gradient of J(k,2) with respect to $\underline{x}_1$ and $\underline{x}_2$, and set it to zero

$$\nabla_{\binom{\underline{x}_1}{\underline{x}_2}} J(k,2) = 0$$

where

$$\underline{x}_j = [x_{1,j}^T, x_{2,j}^T, \ldots, x_{k,j}^T]^T \tag{28}$$

We can obtain a final recursive estimation equation as:

$$\begin{bmatrix} \hat{x}_{k,1|k,2} \\ \hat{x}_{k,2|k,2} \end{bmatrix} = E_{k,2} \begin{bmatrix} P_{x,1}^{-1} \hat{x}_{k,1} \\ H^T R_{k,2}^{-1} m_{k,2} \end{bmatrix} + F_{k,2} \begin{bmatrix} \hat{x}_{k-1,1|k-1,2} \\ \hat{x}_{k-1,1|k-1,2} \end{bmatrix} \tag{29}$$

For notation used in the above equation, see Reference 17. With reference to the final estimation equation above, it is noted that for the filtered estimate $\hat{x}_{k,2|k,2}$ the scheme uses the previous estimates $\hat{x}_{k,1}$, $\hat{x}_{k-1,1|k-1,2}$, and $\hat{x}_{k-1,2|k-1,2}$, and the measurement $m_{k,2}$. Here, it should be emphasized that the scheme uses the smoothed estimate $\hat{x}_{k-1,1|k-1,2}$ instead of $\hat{x}_{k-1,1}$. Thus, when the filtered estimate $\hat{x}_{k,2|k,2}$ is computed, we need to update the estimate $\hat{x}_{k,1}$ to $\hat{x}_{k,1|k,2}$ for use in the next iteration. Now, by using the recursive estimation equation in Eq. (28) and the pseudo error covariance equation in Reference 17, we can compute $\hat{x}_{k,2|k,2}$, k = 2,...,N recursively.

The resultant recursive filtering equation for $\hat{x}_{k,3|k,3}$ becomes similar to the one in Eq. (28). Also, the smoothing equation for $\hat{x}^*_{i,3/i+1,3}$ becomes similar to the one for $\hat{x}^*_{i,2/i+1,2}$. After all the estimates in the third column, $\hat{x}^*_{i,3|k,3}$ and $\hat{x}^*_{k,3|k+1,3}$ for k = 1, 2,...,N are obtained, the smoothed estimates $\hat{x}^*_{k,3|k+1,3}$, k = 1,...,N-1, will be used for the estimates

A QUARTER-PLANE PROCESSOR

The estimate $\hat{x}_{k,\ell|p,q}$ is defined as the estimate of $x_{k,\ell}$ obtained by fitting as approximating function in the region R(p,q) where

$$R_{(p,q)} \overset{\Delta}{=} \{(\xi,\eta) |\ \xi_1 \leq \xi \leq \xi_p \quad , \quad \text{and } \eta_1 \leq \eta \leq \eta_q\} \tag{26}$$

For (p,q) = (k,ℓ), $\hat{x}_{k,\ell|k,\ell}$ becomes a filtered estimate. For $p \geq k$ and $q \geq \ell$, except for p = k and q = ℓ, $\hat{x}_{k,\ell|p,q}$ becomes a smoothed estimate of $x_{k,\ell}$. In our formulation, $\hat{x}_{k,\ell|p,q}$ would be obtained by minimizing the objective function $J_{(p,q)}$ in the region $R_{(p,q)}$:

$$J(p,q) = \sum_{j=1}^{q} \sum_{i=1}^{p} [(Hx_{i,j}-m_{i,j})^T R_{i,j}^{-1} (Hx_{i,j}-m_{i,j})]$$

$$+ [\sum_{j=1}^{q-1} \sum_{i=1}^{p-1} (x_{i,j}{}^T,\ x_{i+1,j}{}^T,\ x_{i,j+1}{}^T,\ x_{i+1,j+1}{}^T)$$

$$\cdot\ C(x_{i,j}{}^T,\ x_{i+1,j}{}^T,\ x_{i,j+1}{}^T,\ x_{i+1,j+1}{}^T)^T] \tag{27}$$

where H = (1,0,0,0).

In a quarter-plane processor, the filtered estimate $\hat{x}_{k,\ell|k,\ell}$ is obtained by using the previous estimates of $x_{k-1,\ell-1}$, $x_{k-1,\ell}$, and $x_{k,\ell-1}$, and the measurement $m_{k,\ell}$. In the next iteration, the filtered estimate $\hat{x}_{k+1,\ell|k+1}$ is obtained by using the previous estimates of $x_{k,\ell-1}$, $x_{k,\ell}$, and $x_{k+1,\ell-1}$ and the measurement $m_{k+1,\ell}$. After estimating all the states in the ℓth column the recursive processor moves to the next column, and estimates $\hat{x}_{k,\ell+1|k,\ell+1}$, k = 2,...,N, and so on. First, we will discuss a filtering procedure for $\hat{x}_{k,2|k,2}$, k = 2, 3,...,N. Then, this procedure is extended to the filtered estimates $\hat{x}_{k,\ell|k,\ell}$ for ℓ = 2, 3,...,M.

so-called "saddle point" and every surface element of such a membrane is "pure twist." An appropriate smoothness measure would be changed to:

$$z(\xi,\eta) = [\frac{\partial^2}{\partial \xi^2} s(\xi,\eta)]^2 + [\frac{\partial^2}{\partial \eta^2} s(\xi,\eta)]^2 \tag{23}$$

3. In Reference 7, Hou and Andrews suggested using $||\nabla^4 s(\xi,\eta)||^2$ as a smoothness measure for a surface. The physical interpretation of the quantity $\nabla^4 s(\xi,\eta)$ is found in a plate bending theory (ref 16); an unloaded plate can bend only in a biharmonic function ω where

$$\nabla^4 \omega = 0 \tag{24}$$

A bicubic Hermite polynomial which minimizes the objective function J is given in Appendix B.

Smoothing Integral

Now, it is needed to determine the function $s(\xi,\eta)$ which minimizes the objective function J in Eq. (16). It is noted that the smoothing integral in its present form gives difficulties in finding an explicit solution. By evaluating the integrals of the derivatives of basis functions and applying some algebraic manipulations, these smoothing integrals are converted to quadratic forms as follows:

$$\rho \int_{\eta_1}^{\eta_M} \int_{\epsilon_1}^{\epsilon_N} [z(\xi,\eta)] d\xi d\eta = \sum_{j=1}^{M-1} \sum_{i=1}^{N-1} \int_{\eta_j}^{\eta_{j+1}} \int_{\xi_i}^{\xi_{i+1}} [z(\xi,\eta)] d\xi d\eta$$

$$= \sum_{j=1}^{M-1} \sum_{i=1}^{N-1} (x_{i,j}{}^T, x_{i+1,j}{}^T, x_{i,j+1}{}^T, x_{2+1,j+1}{}^T) \cdot$$

$$C \cdot (x_{i,j}{}^T, x_{i+1,j}{}^T, x_{i,j+1}{}^T, x_{i,j+1}{}^T)^T \tag{25}$$

where C is a 16 by 16 matrix.

for $i = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, M$. Then, a piecewise bicubic Hermite polynomial is completely defined by $x_{i,j}$, $i = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, M$, as follows:

$$s(\xi,\eta) = s_{i,j}(\xi,\eta) \quad , \quad \text{for } \xi_i \leq \xi \leq \xi_{i+1} \tag{18}$$

and

$$\eta_j \leq \eta \leq \eta_{j+1} \tag{19}$$

where

$$s_{i,j}(\xi,\eta) = \sum_{\ell=0}^{1} \sum_{m=0}^{1} \begin{bmatrix} \phi_\ell(\xi)\cdot\phi_m(\eta) \\ \psi_\ell(\xi)\cdot\phi_m(\eta) \\ \phi_\ell(\xi)\cdot\psi_m(\eta) \\ \psi_\ell(\xi)\cdot\psi_m(\eta) \end{bmatrix}^T \cdot x_{i+\ell,j+m} \tag{20}$$

Choice of the Smoothness Measure $z(\xi,\eta)$

Here, we present three examples of the smoothness measures and compare their physical implications.

1. Gaussian curvature: In Reference 15, the mean curvature of a surface at (ξ,η) is defined as:

$$(0.5)\,\nabla^2 s(\xi,\eta) \tag{21}$$

Noting the Euler's theorem (ref 16) that the sum of two curvatures in perpendicular directions at a point is constant, the square of $\nabla^2 s(\xi,\eta)$ in Eq. (21) would be a reasonable measure for the smoothness of a surface:

$$z(\xi,\eta) = \left[\frac{\partial^2}{\partial\xi^2} s(\xi,\eta) + \frac{\partial^2}{\partial\eta^2} s(\xi,\eta)\right]^2 \tag{22}$$

2. A variation from the Gaussian curvature: With reference to Eq. (22), an interesting case occurs when the two principal curvatures are equal and of opposite sign. The mean curvature in this case is zero. This is the

PROBLEM FORMULATION FOR TWO-DIMENSIONAL APPROXIMATION

When the observation data are noise corrupted and the underlying system is unknown, it is proposed to approximate the original signal by spline functions which minimize a certain objective function. Thus, from a set of discrete measurements $m_{i,j}$ corrupted by white noise process $v_{i,j}$

$$m_{i,j} = f(\xi_i, \eta_j) + v_{i,j} \quad , \quad i = 1, 2, \ldots, N, \; j = 1, 2, \ldots, N \tag{15}$$

The original two-dimensional signal $f(\xi, \eta)$ defined in the region of (ξ, η) is approximated by a spline function $s(\xi, \eta)$ which minimizes the following objective function:

$$J = \sum_{j=1}^{N} \sum_{i=1}^{N} [s(\xi_i, \eta_j) - m_{i,j}]^T R_{i,j}^{-1} [s(\xi_i, \eta_j) - m_{i,j}] + \rho \left[\int_{\eta_1}^{\eta_M} \int_{\xi_1}^{\xi_N} z(\xi, \eta) d\xi d\eta \right] \tag{16}$$

where $\rho > 0$ is the smoothing parameter; $R_{i,j}$ is the observation error covariance; and $z(\xi, \eta)$ is a certain smoothness measure of $s(\xi, \eta)$ at (ξ, η).

Choice of an Approximating Function

In this report, we are interested in obtaining smoothed estimates of function values and the first derivatives in both ξ and η directions. Here, we propose to restrict our approximating functions to piecewise bicubic Hermite polynomials which have continuous first derivatives in both ξ and η directions.

Define

$$x_{i,j} \stackrel{\Delta}{=} \left[s(\xi, \eta) \quad , \quad \frac{\partial s}{\partial \xi}(\xi, \eta) \quad , \quad \frac{\partial s}{\partial \eta}(\xi, \eta), \quad \frac{\partial^2 s}{\partial \xi \partial \eta}(\xi, \eta) \right]^T \Bigg|_{(\xi, \eta) = (\xi_i, \eta_j)} \tag{17}$$

$$m_i = f(\xi_i) + v_i \quad , \quad i = 1,\ldots,100 \tag{12}$$

where v_i is white Gaussian measurement noise,

$$R_i = E\{v_i v_i^T\} = 0.000025, \text{ and } \Delta\xi = \xi_n - \xi_{n-1} = 2\pi/100 = 0.062832 \tag{13}$$

Function values and the first derivatives at discrete nodes are estimated from the measurements m_i, $i = 1,\ldots,100$ by the three schemes below:

1. Difference quotients method.
2. Recursive smoothing algorithm with $\ell = 1$: Eq. (10).
3. Nonrecursive smoothing by cubic splines as described in Reference 6.

Table I shows the mean-square errors from the three schemes above where

$$\varepsilon_0 = \frac{1}{100} \sum_{i=1}^{100} (f(\xi_i) - \hat{x}_{i|i+1}(1))^2,$$

$$\varepsilon_1 = \frac{1}{100} \sum_{i=1}^{100} (f'(\xi_i) - \hat{x}_{i|i+1}(2))^2 \tag{14}$$

TABLE I. MEAN-SQUARE ERRORS

	Difference Quotients	Recursive Smoothing by using Eq. (10)	Nonrecursive Smoothing by Cubic Splines, Ref. 6
ε_0	$2.5\text{x}10^{-5}$	$0.8\text{x}10^{-5}$	$0.57\text{x}10^{-5}$
ε_1	$4.30\text{x}10^{-1}$	$0.12\text{x}10^{-1}$	$0.1007\text{x}10^{-1}$

From Table I, it is noted that both smoothing algorithms are successful in reducing the error in the estimated states. The error in the first derivative is decreased by more than 10 db. Moreover, Table I shows that the performance of the two smoothing schemes are comparable. However, it should be emphasized that the recursive algorithm developed in this report is much simpler than the nonrecursive spline smoothing.

following. The smoothed estimate of x_i obtained by fitting cubic splines to m_n, $n = 2, 3, \ldots, i+\ell$ with the initial values $\hat{x}_1$ and $E_1 = \rho_1$ is the same as the smoothed estimate of x_i obtained by fitting splines to m_n, $n = i+1, \ldots, i+\ell$ with the filtered estimate $\hat{x}_{i|i}$ and E_i.

A recursive procedure to obtain the smoothed estimate $\hat{x}_{i|i+1}$ is summarized as follows:

Part 1. Obtain the filtered estimates $\hat{x}_{i|i}$, $i = 2, \ldots, N$ by using Eqs. (7d), (9), and (9a).

Part 2. Obtain smoothed estimates $\hat{x}_{i|i+1}$, for $i = 1, \ldots, N-1$ by using Eqs. (10), (10a), and (10b).

As was mentioned earlier in this section, the smoothed estimate $\hat{x}_{i|i+\ell}$ is an approximation to the nonrecursive solution $\hat{x}_j{}^* = \hat{x}_{i|N}$. Thus, as the number of delays, ℓ, increases, we will get a better approximation to $x_i{}^*$. For $\ell = 2, 3, \ldots$, only the smoothing part is modified by solving the simultaneous equations in Eq. (7) with $k = i+\ell$. In fact, the smoothing algorithm developed by far is a fixed-lag smoothing algorithm, which is suitable for an on-line implementation. If the situation does not require an on-line implementation, we can also derive a fixed-interval smoothing algorithm with observation set $M = \{m_1, \ldots, m_N\}$. In this case, the resultant smoothed estimates become exactly the same as the nonrecursive estimates.

SIMULATION RESULTS

For a continuous signal

$$f(\xi) = \sin(\xi) \tag{11}$$

Measurements are obtained at discrete points:

With reference to Eqs. (7) and (8), each iteration of the recursive filtering algorithm can be interpreted as fitting a cubic polynomial to the previous estimate $\hat{x}_{i-1|i-1}$ and the present measurement m_i in the region $[\xi_{i-1},\xi_i]$.

Smoothing

A smoothed estimate $\hat{x}_{i|i+\ell}$ is defined as the estimate of x_i obtained by solving the minimization problem in Eq. (6) with $N = i+\ell$. In fact, this can be interpreted as fitting a polynomial spline to the first $i+\ell$ data and obtaining the function value and its derivative from the approximating function at the node i. In our formulation, this corresponds to solving the simultaneous equations of the same form as Eq. (7) with $j = 1$, $k = i+\ell$, and with the quantities G_i and d_i defined in Eqs. (7a) and (7b). By using the same reduction method as before, the result would be the same form as itself (Eq. (7) with $j = 1$ and $k = i+\ell$).

For the case of a one-sample delay, $x_{i|i+1}$ is obtained by solving the simultaneous equations in Eq. (7) with $j = 1$, $k = i+1$, which, in turn, yields

$$\hat{x}_{i|i+1} = V_i E_i^{-1} \hat{x}_{i|i} + K_i m_{i+1} \tag{10}$$

where

$$V_i = [-\rho_i C_{12}(\rho_i C_{22} + H^T R_{i+1}^{-1} H)\rho_i C_{21} + G_i]^{-1} \tag{10a}$$

$$K_i = -V_i \rho_i C_{12}(\rho_i C_{22} + H^T R_{i+1}^{-1} H)^{-1} H^T R_{i+1}^{-1} m_{i+1} \tag{10b}$$

and E_i is defined as before.

Equation (10) is the desired smoothing algorithm, in which the smoothed estimate $\hat{x}_{i|i+1}$ is obtained by updating the filtered estimate $x_{i|i}$ with the measurement m_{i+1}. The smoothing procedure described above implies the

for x_i. Thus, by eliminating the first four equations, i.e., the variable x_1, Eq. (7) is reduced to the same form as itself with $j = 2$, $k = i$, and $\hat{x}_{2|2}$ and E_2 are calculated by Eqs. (7c) and (7d). Note that the quantity E_j defined by Eq. (7d) is called pseudo error covariance, because $P_{k|k} = E\{(x_k-\hat{x}_{k|k})(x_k-\hat{x}_{k|k})^T\}$ cannot be computed in a recursive manner directly.

By applying this reduction method repeatedly, the original equations in Eq. (7) are reduced to the same form as themselves with $j = i-1$, $k = 1$ and every $x_{j|j}$ and E_j is computed by Eqs. (7c) and (7d) recursively. Solving Eq. (7) with $j = i=1$, $k = i$ in terms of x_i, we obtain a recursive estimate algorithm as

$$\hat{x}_{i|i} = E_i[H^T R_i^{-1} m_i - \rho_{i-1} C_{21} G_{i-1}^{-1} d_{i-1}] \tag{8}$$

Equation (8) is rearranged as

$$\hat{x}_{i|i} = E_i H^T R_i^{-1} m_i + F_i \hat{x}_{i-1|i-1} \tag{9}$$

where

$$F_i = -\rho_{i-1} E_i C_{21} (\rho_{i-1} C_{11} + E_{i-1}^{-1})^{-1} E_{i-1}^{-1} \tag{9a}$$

and E_i's are computed by Eq. (7d) recursively. Equation (9) above is the desired filtering equation which computes $\hat{x}_{i|i}$ from the previous estimate $\hat{x}_{i-1|i-1}$ and the present measurement m_i. From the viewpoint of smoothing splines, the recursive filtering algorithm can be interpreted as follows. The estimate of x_i obtained by fitting cubic splines to the measurement data m_n, $n = 2, 3, \ldots$, with the initial values $\hat{x}_1$ and $E_1 = P_1$ is the same as the estimate of x_i obtained by fitting a cubic polynomial to the measurement m_i with the initial values at stage $i-1$, $\hat{x}_{i-1|i-1}$, and E_{i-1}. In fact, the above interpretation comes from the mathematical derivations in Eq. (7) through Eq. (9).

$$
\begin{bmatrix}
G_j & \rho_j C_{12} & & & \underline{0} \\
\rho_j C_{21} & \begin{array}{l}\rho_j C_{22} \\ + \rho_{j+1} C_{11} \\ + H^T R_{j+1}^{-1}\end{array} & \rho_{j+1} C_{12} & & \\
 & + \rho_{j+1} C_{21} & \ddots & \rho_{1-2} C_{12} & \\
 & \ddots & \rho_{k-2} C_{21} & \begin{array}{l}\rho_{k-2} C_{22} \\ + \rho_{k-1} C_{11} \\ + H^T R_{k-1}^{-1}\end{array} & \rho_{k-1} C_{12} \\
\underline{0} & & & \rho_{k-1} C_{21} & \begin{array}{l}\rho_{k-1} C_{22} \\ + H^T R_k^{-1} H\end{array}
\end{bmatrix}
\cdot
\begin{bmatrix}
x_j \\ x_{j+1} \\ \vdots \\ x_{k-1} \\ x_k
\end{bmatrix}
=
\begin{bmatrix}
d_j \\ H^T R_{j+1}^{-1} m_{j+1} \\ \vdots \\ H^T R_{k-1}^{-1} m_{k-1} \\ H^T R_k^{-1} m_k
\end{bmatrix}
\tag{7}
$$

where $j = 1$ and $k = i$, and G_j and d_j are defined by

$$G_j \overset{\Delta}{=} \rho_j C_{11} + E_j^{-1} \tag{7a}$$

$$d_j \overset{\Delta}{=} E_j^{-1} \hat{x}_{j|j} \tag{7b}$$

$$\hat{x}_{j|j} = E_j[H^T R_j^{-1} m_j + \rho_{j-1} C_{21} G_{j-1}^{-1} d_{j-1}], \quad \hat{x}_{1|1} \overset{\Delta}{=} \hat{x}_1 \tag{7c}$$

$$E_j^{-1} = \rho_{j-1} C_{22} + H^T R_j^{-1} H - (\rho_{j-1} C_{21}) G_{j-1}^{-1} (\rho_{j-1} C_{12}), \quad E_1 \overset{\Delta}{=} P_1 \tag{7d}$$

It should be noted that the matrix on the left side of Eq. (7) is diagonally dominant and positive definite. Here, we are interested in solving Eq. (7)

$$J_N = \sum_{n=2}^{N} [(Hx_n - m_n)^T R_n^{-1}(Hx_n - m_n)] + (x_1 - \hat{x}_1)^T P_1^{-1}(x_1 - \hat{x}_1)$$

$$+ \sum_{n=2}^{N} \rho_n (x_n - A^* x_{n-1})^T B^{-1}(x_n - A^* x_{n-1}) \quad (6)$$

where H = (1 0).

Let the solutions to the above optimization problem be $[\hat{x}_1^*, \hat{x}_2^*, \ldots, \hat{x}_N^*]$. If $\hat{x}_{p|q}$ is defined as the estimate of x_p obtained by minimizing Eq. (6) with N = q, then $\hat{x}_i^*$ can be written as $\hat{x}_{i|N}$. Here, it is proposed to approximate a nonrecursive solution $\hat{x}_i^* = \hat{x}_{i|N}$ by $\hat{x}_{i|i+\ell}$, where $\ell = 0$ and $\ell = N$. As has been mentioned before, due to a local base property of the polynomial splines, the smoothed estimate $\hat{x}_{i|i+\ell}$ would be sufficiently close to the nonrecursive solution $\hat{x}_i^*$ for $\ell = 1$, 2, or 3.

Filtering

From the definition, a filtered estimate $\hat{x}_{i|i}$ would be obtained by minimizing the objective function in Eq. (6) with N = i. By taking the gradient of J_i with respect to $[x_1, x_2, \ldots, x_i]$, we have a set of simultaneous equations as follows:

rover. It is desired to calculate the cross-path slope at point (β_i, θ_j). However, in general, $\rho_{i,j} \neq \rho_{i,j+1}$. Thus, the true cross-path slope is not along an arc connecting points $\rho_{i,j}$ and $\rho_{i,j+1}$.

Our algorithm to calculate the terrain cross-path slope can then be summarized as follows:

1. Obtain the range measurements.

2. Use the smoothing algorithm to calculate the range cross-path slope.

3. Obtain the terrain cross-path slope and its variance.

<u>Evaluation of Terrain Variables</u>

As mentioned in the introduction, in the evaluation of terrain variables, we only use the slopes at the spine and track points. The reason is twofold. First, only a minor part of this path selection scheme needs the data of elevation. Second, if we adopt the elevation estimates as our input data, we will get larger errors in the calculation of in-path and tilt slope terrain variables.

The terrain variables and the variances together with the corresponding explanations are listed below (ref 20).

1. <u>In-Path Terrain Variables</u>. The in-path slope terrain variable gives the average of the in-path slopes for the four vehicle wheels at each section. This variable is a measure of the risk in the forward direction.

2. <u>Tilt Slope</u>. Tilt slope terrain variable is used to estimate excessive cross-path slopes which may cause the vehicle to tip over.

3. <u>Obstruction Height</u>. The obstruction height is calculated for six different locations at each discrete section of the terrain. The maximum value is then chosen as representative of this whole section.

In deriving the formula for a typical obstruction height, we use a third order polynomial to approximate the terrain elevation in each location. By differentiating this polynomial with respect to the distance, we get an expression for the slope. With the known data of slopes at the three points substituted into this expression, we can determine the coefficients of the polynomial. Using this polynomial, we can then find the obstruction height in this direction.

4. Wheel Deviation. The wheel deviation variable describes the offset of any of the four wheels from a plane. Wheels on any three track points define a plane. For each combination of three wheels touching the terrain, the deviation of the fourth wheel with respect to this plane is defined as the wheel deviation.

A set of the measurement data is obtained by the described scanning scheme. The range measurement data are processed by the gradient estimation scheme to evaluate in-path and cross-path slopes at the data points. Since the slopes are estimated in the spherical coordinate system, it is necessary to transform the range slope in the spherical coordinate system to terrain slope in the cylindrical coordinate system. The in-path and cross-path slopes and their covariances at the spline and track points along the corridors are evaluated by applying a two-dimensional interpolation scheme over the estimated slopes at the data points. The terrain variables at a discrete section along each corridor are computed by using estimated slopes at the spline and track points. Since the terrain variable estimates have uncertainty, the present method increases the reliability by considering standard deviation as well as mean values.

CONCLUSION

By taking an algebraic approach, a recursive smoothing algorithm was developed as an approximation to nonrecursive spline smoothing. Compared to the recursive smoothing algorithm suggested by Weinert, the smoothing algorithm in this report is simpler in that the scheme is in a discrete form. Simulation result shows that the performance of the recursive smoothing algorithm is comparable to that of its nonrecursive counterpart. In addition, the computational complexity with recursive smoothing algorithm is much less than its nonrecursive one. Also, recursive smoothing by splines can be implemented on-line. By taking an algebraic approach, a two-dimensional recursive smoothing algorithm was developed as an approximation to a nonrecursive smoothing spline technique. While the amount of computation required for a nonrecursive algorithm increases rapidly with the size of the two-dimensional data, the amount of computation for this smoothing algorithm increases only linearly.

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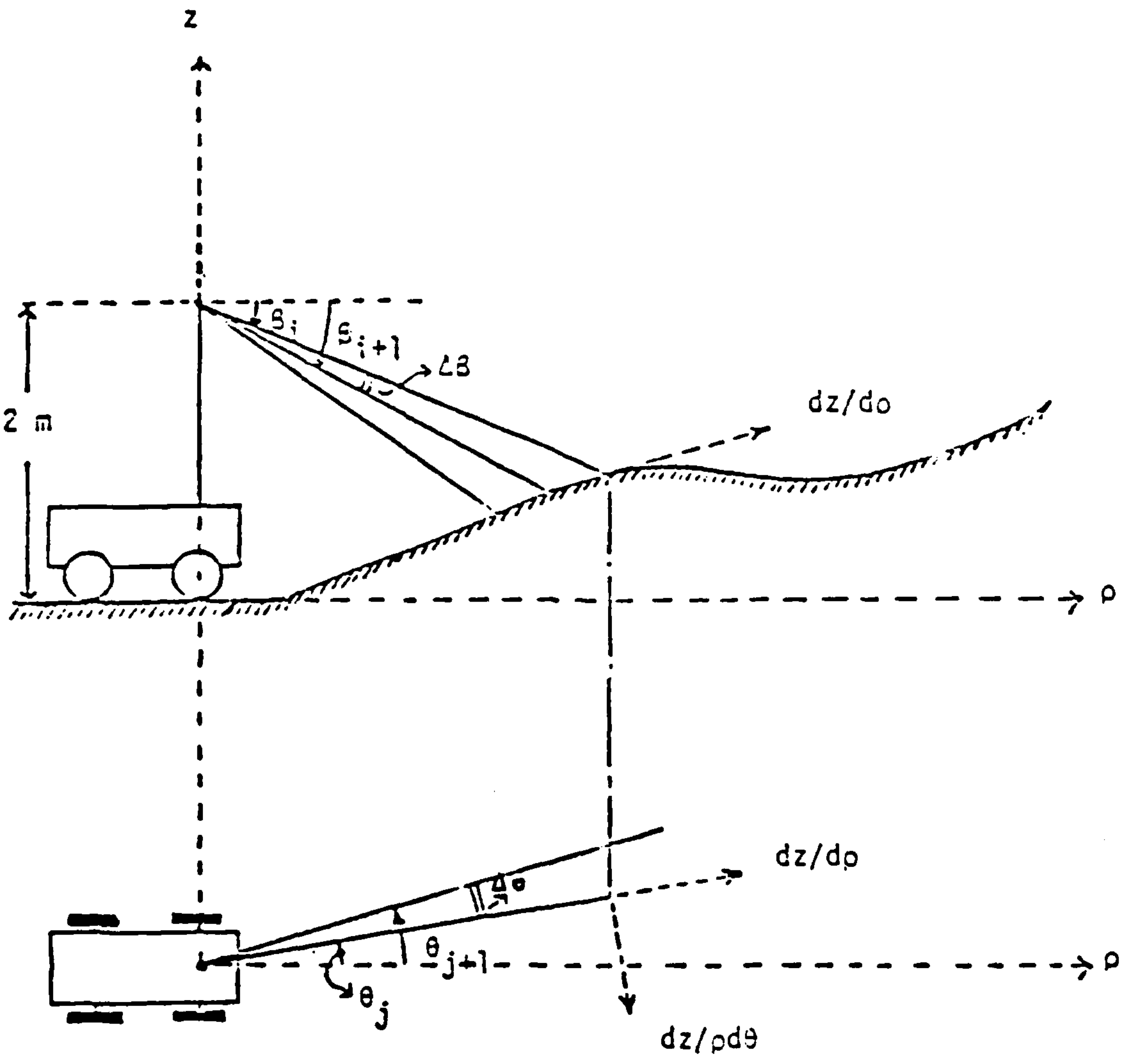

Figure 1. Top and side views of a rangefinder.

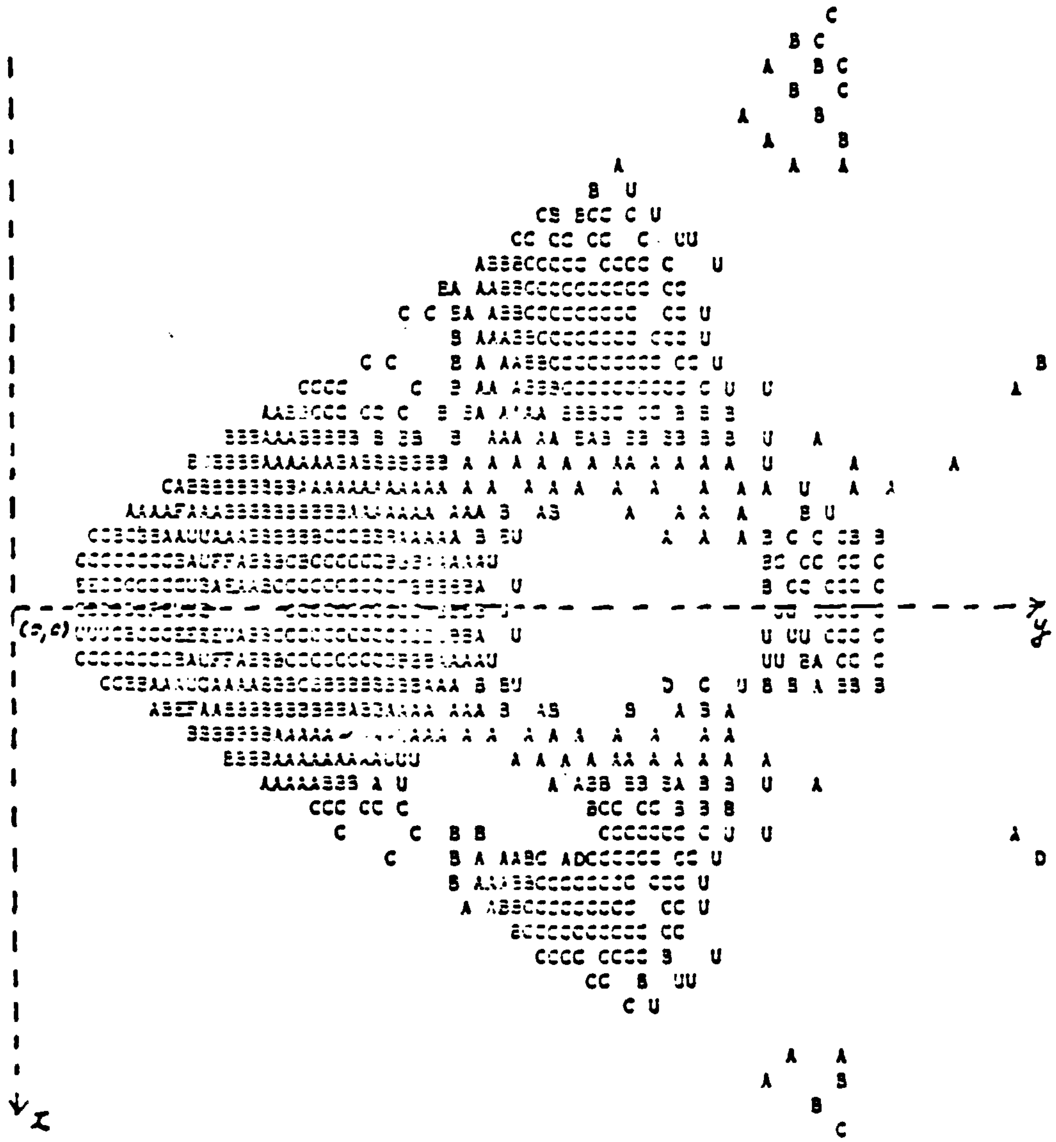

Figure 2. A slope map in the x-y plane for the in-path terrain slopes.

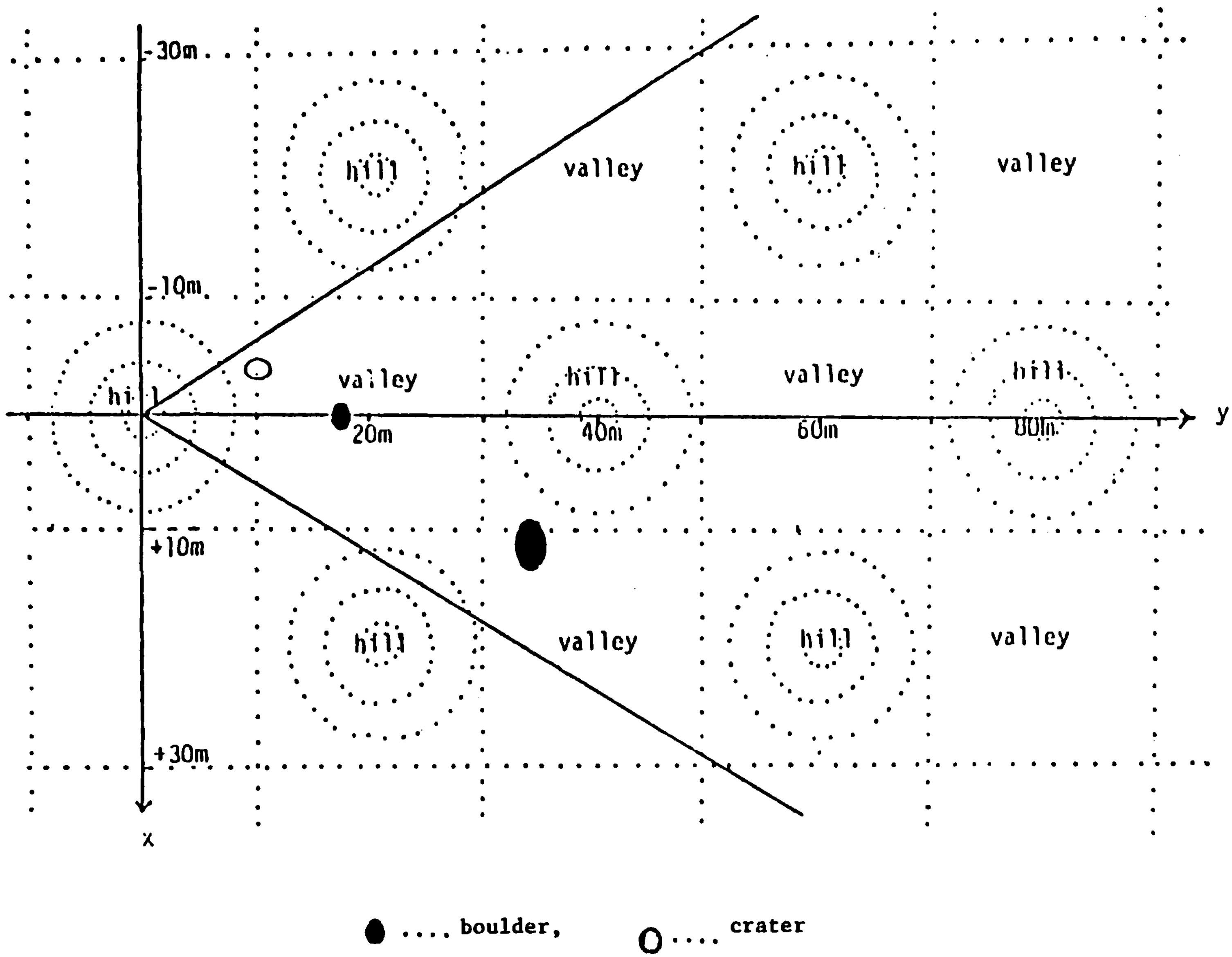

Figure 3. Top view of the terrain model used for simulation.

APPENDIX A

EVALUATION OF SMOOTHING INTEGRALS

From Eqs. (2a) through (2e) of the text, a piecewise cubic Hermite polynomial in the section $[\xi_{i-1},\xi_i]$ is represented as:

$$s_{i-1,i}(\xi) = \begin{bmatrix} \phi_{i,1}(\xi) \\ \psi_{i,1}(\xi) \\ \phi_{i,0}(\xi) \\ \psi_{i,0}(\xi) \end{bmatrix}^T \cdot \begin{bmatrix} s(\xi_i) \\ s'(\xi_i) \\ s(\xi_{i-1}) \\ s'(\xi_{i-1}) \end{bmatrix} \tag{A-1}$$

where $s(\xi_{i-1})$, $s'(\xi_{i-1})$, $s(\xi_i)$, and $s'(\xi_i)$ are the function values and first derivatives at the nodes i-1, and i. We make the change of variables such as

$$\mu = \xi - \xi_{i-1} \tag{A-2}$$

This change of variables does not affect the value of the smoothing integral and results in a simpler computation. The smoothing integral in the interval $[\xi_{i-1},\xi_i]$ is

$$I_{i-1,j} = \int_{\xi^+_{i-1}}^{\xi^-_i} ||s''_{i-1,j}(\xi)||^2 \, d\xi = \int_{0^+}^{\Delta^-} ||s''_{i-1,j}(\mu)||^2 d\mu \tag{A-3}$$

where

$$\Delta = \mu_i - \mu_{i-1} = \xi_i = \xi_{i-1}$$

Using Eq. (A-2) and Eq. (2a) of the text, Eq. (A-1) becomes

$$s_{i-1,i}(\mu) = [\phi_{i,1}(\mu)\psi_{i,1}(\mu) \;\; \phi_{i,0}(\mu) \;\; \psi_{i,0}(\mu)][x_i^T \; x_{i-1}^T]^T \tag{A-4}$$

Thus, the norm square of the second derivative is written as:

$$||s''_{i-1,i}(\mu)||^2 = [x_i^T \; x_{i-1}^T]^T \cdot$$

$$\begin{bmatrix} \phi''_{i,1}(\mu)\phi''_{i,1}(\mu), & \phi''_{i,1}(\mu)\psi''_{i,1}(\mu), & \phi''_{i,1}(\mu)\phi''_{i,0}(\mu), & \phi''_{i,1}(\mu)\psi''_{i,0}(\mu) \\ \psi''_{i,1}(\mu)\phi''_{i,1}(\mu), & \psi''_{i,1}(\mu)\psi''_{i,1}(\mu), & \psi''_{i,1}(\mu)\phi''_{i,0}(\mu), & \psi''_{i,1}(\mu)\psi''_{i,0}(\mu) \\ \phi''_{i,0}(\mu)\phi''_{i,1}(\mu), & \phi''_{i,0}(\mu)\psi''_{i,1}(\mu), & \phi''_{i,0}(\mu)\phi''_{i,0}(\mu), & \phi''_{i,0}(\mu)\psi''_{i,0}(\mu) \\ \psi''_{i,0}(\mu)\phi''_{i,1}(\mu), & \psi''_{i,0}(\mu)\psi''_{i,1}(\mu), & \psi''_{i,0}(\mu)\phi''_{i,0}(\mu), & \psi''_{i,0}(\mu)\psi''_{i,0}(\mu) \end{bmatrix} \cdot$$

$$\cdot \begin{bmatrix} x_i \\ x_{i-1} \end{bmatrix} = \begin{bmatrix} x_i \\ x_{i-1} \end{bmatrix}^T \cdot \begin{bmatrix} k_{i-1,i}(\mu) \end{bmatrix} \cdot \begin{bmatrix} x_i \\ x_{i-1} \end{bmatrix} \tag{A-5}$$

By utilizing Eq. (A-5), Eq. (A-3) becomes:

$$I_{i-1,i} = \int_{0^+}^{\Delta^-} ||s''_{i-1,i}(\mu)||^2 d\mu = \begin{bmatrix} x_i \\ x_{i-1} \end{bmatrix}^T \cdot \begin{bmatrix} \int_{0^+}^{\Delta^-} K_{i-1,i}(\mu) d\mu \end{bmatrix} \cdot \begin{bmatrix} x_i \\ x_{i-1} \end{bmatrix} \tag{A-6}$$

Thus, smoothing integral is obtained as:

$$I_{i-1,1} = \begin{bmatrix} x_i \\ x_{i-1} \end{bmatrix}^T \cdot \begin{bmatrix} 12\Delta^{-3} & -6\Delta^{-2} & -12\Delta^{-3} & -6\Delta^{-2} \\ -6\Delta^{-2} & 4\Delta^{-1} & 6\Delta^{-2} & 2\Delta^{-1} \\ -12\Delta^{-3} & 6\Delta^{-2} & 12\Delta^{-3} & 6\Delta^{-2} \\ -6\Delta^{-2} & 2\Delta^{-1} & 6\Delta^{-2} & 4\Delta^{-1} \end{bmatrix} \cdot \begin{bmatrix} x_i \\ x_{i-1} \end{bmatrix} \tag{A-7}$$

By defining B^{-1} and A^* as below:

$$B^{-1} = \begin{bmatrix} 12\Delta^{-3} & -6\Delta^{-2} \\ -6\Delta^{-2} & 4\Delta^{-1} \end{bmatrix} \qquad A^* = \begin{bmatrix} 1 & \Delta \\ 0 & 1 \end{bmatrix} \tag{A-8}$$

Equation (A-7) is rewritten in the following form as:

$$I_{i-1,i} = \begin{bmatrix} x_i \\ x_{i-1} \end{bmatrix}^T \cdot \begin{bmatrix} B^{-1} & -B^{-1}A^* \\ -A^{*T}B^{-1} & A^{*T}B^{-1}A^* \end{bmatrix} \cdot \begin{bmatrix} x_i \\ x_{i-1} \end{bmatrix} \tag{A-9}$$

$$= \begin{bmatrix} x_{i-1} \\ x_i \end{bmatrix}^T \cdot \begin{bmatrix} A^{*T}B^{-1}A^* & -A^{*T}B^{-1} \\ -B^{-1}A^* & B^{-1} \end{bmatrix} \cdot \begin{bmatrix} x_{i-1} \\ x_i \end{bmatrix} \tag{A-10}$$

$$= (x_i - A^*x_{i-1})^T B^{-1}(x_i - A^*x_{i-1}) \tag{A-11}$$

which is Eq. (3) in the text.

APPENDIX B

In this Appendix, we show that a piecewise bicubic Hermite polynomial which minimizes the objective function in Eq. (B-1) becomes a bicubic spline.

$$J = J_E + \rho J_s \tag{B-1}$$

where

$$J_E = \sum_{j=1}^{M} \sum_{i=1}^{N} [s(\xi_i, \eta_j) - m_{i,j}]^T R_{ij}^{-1} [s(\xi_i, \eta_j) - m_{i,j}] \tag{B-2}$$

and

$$J_s = \int_{\eta_1}^{\eta_M} \int_{\xi_1}^{\xi_N} \left(\frac{\partial^4}{\partial \xi^2 \partial \eta^2} s(\xi, \eta)\right)^2 d\xi d\eta \tag{B-3}$$

Let a set S be a collection of all piecewise bicubic Hermite polynomials. Also, we define a set U as a collection of all piecewise bicubic Hermite polynomials which satisfies constraints set D in Eq. (B-4).

$$\begin{aligned}
&s(\xi_i,\eta_j) = c(i,j) && , \quad i = 1,2,\ldots,N \text{ and } j = 1,2,\ldots,M \\
&\partial s(\xi_i,\eta_j)/\partial \xi = c_\zeta(i,j) && , \quad j = 1,2,\ldots,M \text{ and } i = 1,N \\
&\partial s(\xi_i,\eta_j)/\partial \eta = c_\eta(i,j) && , \quad i = 1,2,\ldots,N \text{ and } j = 1,M \\
&\partial^2 s(\xi_i,\eta_j)/\partial \xi \partial \eta = c_{\xi,\eta}(i,J) && , \quad i = 1,N \text{ and } j = 1,M
\end{aligned} \tag{B-4}$$

Then the minimizing problem in Eq. (A-1) is rewritten as:

$$\min_{s(\xi,\eta)\epsilon S} J = \min_{s(\xi,\eta)\epsilon S} [J_E + \rho J_s] = \min_{D} [J_E + \rho \min_{s(\xi,\eta)\epsilon U} J_s] \tag{B-5}$$

In the paper by DeBoor (ref B-1), it is noted that there exists a unique bicubic spline $g(\xi,\eta)$ in the set U. Also, by using a standard technique to derive the minimum norm property (ref B-2) of a bicubic spline, it can be shown that:

$$J_s = \int_{\eta_1}^{\eta_M} \int_{\xi_1}^{\xi_N} \left(\frac{\partial^4 g(\xi,\eta)}{\partial \xi^2 \partial \eta^2}\right)^2 d\xi d\eta \tag{B-6}$$

Since the bicubic spline $g(\xi,\eta)$ is unique, we have the following Lemma:

Lemma 1: A bicubic Hermite polynomial $s(\xi,\eta) \epsilon U$ which minimizes the smoothing integral J_s, becomes a bicubic spline $g(\xi,\eta)$.

With reference to Eq. (B-5) and Lemma 1, we conclude that a piecewise bicubic Hermite polynomial $s(\xi,\eta)$, which minimizes Eq. (B-5), becomes a cubic spline.

APPENDIX REFERENCES

B-1. DeBoor, C., "Bibubic Spline Interpolation," J. Math. Phys., Vol. 41, 1962, pp. 212-218.

B-2. Ahlberg, J. H., Nilson, E. N., and Walsh, J. L., The Theory of Splines and Their Application, Academic Press, Inc., 1967.

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